2. ZHUR D.S. and PERLIN P.I., Estimation of the accuracy of the solution of problems in the theory of elasticity near irregular points of the boundary. Aerofizika i Geokosmicheskiye Issledovaniya, MFTI, Moscow, 1984.
3. KONDRAT'YEV V.A., Boundary-value problems for elliptic equations in regions with conical or angular points. Tr. Mosk. Mat. O-va, 16, 1967.
4. WILLIAMS M.L., Stress singularities resulting from various boundary conditions in angular corners of plates in extension. J. Appl. Mech. 19, 4, 1952.
5. KALANDIYA A.I., Remarks on the singularity of elastic solutions near corners. PMM, 33, 1, 1969.
6. MAZ'YA V.G. and PLAMENEVSKII B.A., On the coefficients in the asymptotic form of the solutions of elliptic boundary-value problems near conical points. Dokl. Akad. Nauk SSSR, 219, 2, 1974.
7. STERN M., A boundary integral representation for stress intensity factors. Proc. 10 Anniv. Meeting of Soc. of Eng. Sci, Raleigh N.C., 1973.
8. STERN M., BECKER E.B. and DUNHAM R.S., A contour integral computation of mixed-made stress intensity factors. Intern. J. Fracture. 12, 3, 1976.
9. KUPRADZE V.D., Potential Methods in the Theory of Elasticity. Fizmatgiz, Moscow, 1963.
10. PERLIN P.I., A numerical method of solving the singular integral equations of the basic three-dimensional problems of the theory of elasticity. Izv. Akad. Nauk SSSR, MTT, 3, 1975.

Translated by L.K.

J. Appl. Maths Mechs, Vol. 55, No. 4, pp. 549-554, 1991
Printed in Great Britain

0021-8928/91 $15.00+0.00

A PSEUDOMACROCRACK IN AN ANISOTROPIC BODY*

V.V. TVARDOVSKII

Moscow

(Received 13 July 1990)

A pseudomacrocrack representing the usual crack in a composite material or in an inhomogeneous body is considered. The crack edges are pulled together by the non-disintegrated elements of the structure, and interact linearly. It is shown that in this case normal extension is sufficient for a non-zero value of the stress intensity factor K_{II} to occur at its tip. The problem is reduced to the Prandtl integro-differential vector equation for which an analytic solution is obtained. A relation is derived connecting the stress intensity factors K_I, K_{II}, with the stiffness of the joints between the edges, the elastic characteristics of the surrounding material and the external loads.

A classical example of a peseudomacrocrack is the macrocrack in a composite material consisting of a brittle ceramic matrix and tensile fibres pulling its edges together and preventing it from enlarging /1, 2/. A model of a pseudomacrocrack in an elastic, linearly anisotropic body in conditions of plane deformation was discussed in /3/. Here the relation between the stresses transmitted from the edge to the edge σ_{ni} and the opening of the edges w_j, was assumed to be linear:

$$\sigma_{ni}(\mathbf{x}) = k_{ij} w_j(\mathbf{x}) \tag{0.1}$$

where k_{ij} is a symmetric tensor. When the tensor k_{ij} is diagonal and the plane of elastic symmetry coincides with the plane of the pseudomacrocrack, the problem splits into two

**Prikl.Matem.Mekhan.*,55,4,685-690,1991

independent problems of symmetric loading and skew-symmetric shear. The following coupling parameter was introduced:

$$\lambda = 2k^2\varkappa \frac{1-\nu_{23}\nu_{32}}{E_{22}}, \quad \varkappa = \operatorname{Re}\left(i\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right) \tag{0.2}$$

whose dimension is the reciprocal of length (the quantity λ^{-1} is identical in its order of magnitude with the characteristic dimension of the material structure) such, that from condition $\lambda l \gg 1$ (l is the length of the pseudomacrocrack) it follows that the stress intensity factor K_I at the tip of the pseudomacrocrack is independent /3/: $K_I = \sigma_y^\infty \sqrt{2/\lambda}$. In formula (0.2) ν_{ij}, E_{22} represent the technical elastic constants and μ_1, μ_2 are the unequal roots of the characteristic Lekhnitskii equation /4/ with a positive imaginary part. An analogous problem for an isotropic body was discussed in /5/ for the range of values $\lambda l \infty 1$.

When the orientation of the pseudomacrocrack in an anisotropic medium is arbitrary, the problem becomes much more complicated. In particular, even the application of normal tensile stresses only results, in addition to the quantity K_I acting at the tip of the pseudomacrocrack, in the appearance of a non-zero value of the quantity K_{II}. A semi-infinite pseudomacrocrack is discussed below, and the relation between the applied forces and the stress intensity factors at its tip is determined.

1. Formulation of the problem, and the resolving equation. Let an infinite anisotropic space be weakened by a semi-infinite pseudomacrocrack lying on the positive part of the x-axis and let it have an elastic plane of symmetry normal to the z-axis of the (x, y, z) system of coordinates. Then the generalized plane problem will split into two independent problems, namely that of plane deformation and that of longitudinal shear /4/. We shall limit ourselves to the plane deformation case $(\varepsilon_z = 0)$ as being the most interesting one from the practical point of view. The stresses $\sigma_x, \sigma_y, \sigma_{xy}$ and displacements u, v are expressed, in the case of different complex roots $\mu_r, (r = 1, 2)$ of the characteristic equation, by two complex Lekhnitskii potentials $(p_r, q_r$ are complex parameters /4/

$$\begin{gathered}\sigma_x = 2\operatorname{Re}(\mu_1^2\Phi_1(z_1) + \mu_2^2\Phi_2(z_2)), \quad \sigma_y = 2\operatorname{Re}(\Phi_1(z_1) + \Phi_2(z_2)) \\ \sigma_{xy} = -2\operatorname{Re}(\mu_1\Phi_1(z_1) + \mu_2\Phi_2(z_2)) \\ u = 2\operatorname{Re}(p_1\varphi_1(z_1) + p_2\varphi_2(z_2)), \quad v = 2\operatorname{Re}(q_1\varphi_1(z_1) + q_2\varphi_2(z_2)) \\ (z_r = x + \mu_r y, \ \Phi_r(z_r) \equiv \varphi_r'(z_r), \ r = 1, 2)\end{gathered} \tag{1.1}$$

The following homogeneous stress field is specified at infinity:

$$\sigma_x^\infty = 0, \quad \sigma_y^\infty = p_\infty, \ \sigma_{xy}^\infty = q_\infty \tag{1.2}$$

A relation connecting the stresses and the jump in displacements of the type (0.1) are given at the edges of the pseudomacrocrack:

$$\sigma_y^\pm(x) = k_2[v(x)], \quad \sigma_{xy}^\pm(x) = k_1[u(x)], \quad x > 0 \tag{1.3}$$

Generalization to more general type couplings encounters no difficulties. The functions $\varphi_r(z_r)$ which are analytic everywhere in the complex z_r plane, except at positive values of the real axis, can be written in the form of Cauchy-type integrals

$$\varphi_r(z) = \Gamma_r z + \frac{1}{2\pi}\int_0^{+\infty} \frac{\mu_t f(x) + g(x)}{\mu_t - \mu_r}\frac{dx}{x-z} \tag{1.4}$$

Here and henceforth the subscripts r and t take the values 1 and 2, alternating with each other. If $r = 1$, then $t = 2$ and vice versa.

The constants Γ_r in (1.4) determine the field (1.2) at infinity, and $f(x)$ and $g(x)$ are positive functions to be determined.

Using (1.1) and (1.4) we obtain the opening of the crack $[u(x)]$ and $[v(x)]$. In the most general case we have

$$[u(x)] = a_{11}g(x) + a_{12}f(x) \quad [v(x)] = a_{21}g(x) + a_{22}f(x) \tag{1.5}$$

$$\begin{aligned}a_{11} &= 2\operatorname{Re}\left(i\frac{p_1-p_2}{\mu_2-\mu_1}\right), \quad a_{12} = 2\operatorname{Re}\left(i\frac{\mu_2 p_1-\mu_1 p_2}{\mu_2-\mu_1}\right) \\ a_{21} &= 2\operatorname{Re}\left(i\frac{q_1-q_2}{\mu_2-\mu_1}\right), \quad a_{22} = 2\operatorname{Re}\left(i\frac{\mu_2 q_1-\mu_1 q_2}{\mu_2-\mu_1}\right)\end{aligned} \tag{1.6}$$

Using the properties of the complex parameters p_r, q_r /6/, we can show that the following relations hold:

$$a_{12} = -a_{21}, \quad a_{11} > 0, \quad a_{22} > 0, \quad \det [a_{ij}] > 0 \tag{1.7}$$

Thus the functions $f(x)$ and $g(x)$ are proportional to the displacement jumps and should therefore vanish as $x \to +0$:

$$g(0) = 0, \quad f(0) = 0 \tag{1.8}$$

Taking relations (1.8) into account, we can write the derivatives of the complex potentials $\varphi_r(z)$ in the form

$$\Phi_r(z) = \Gamma_r + \frac{1}{2\pi} \int_0^{+\infty} \frac{\mu_t f'(x) + g'(x)}{\mu_t - \mu_r} \frac{dx}{x - z} \tag{1.9}$$

while the stress distribution on the x-axis is given by the formulas

$$\sigma_y^{\pm}(x) + i\sigma_{xy}^{\pm}(x) = p_\infty + iq_\infty + \frac{1}{\pi} \int_0^{+\infty} \frac{f'(\xi) + ig'(\xi)}{\xi - x} d\xi \tag{1.10}$$

while the integral in (1.10) is regarded, for positive values of x, as its principal Cauchy value.

Using relations (1.6) and (1.10) to satisfy boundary conditions (1.3) of the problem, we obtain a singular integrodifferential equation for the unknown vector function $\mathbf{f}(x)$:

$$\Lambda \mathbf{f}(x) - \frac{1}{\pi} \int_0^{+\infty} \frac{\mathbf{f}'(\xi)}{\xi - x} d\xi = \mathbf{P}_\infty; \quad \mathbf{f}(x) = \begin{Bmatrix} g(x) \\ f(x) \end{Bmatrix}, \quad \mathbf{P}_\infty = \begin{Bmatrix} q_\infty \\ p_\infty \end{Bmatrix} \tag{1.11}$$

Here the components of the matrix Λ can be found in terms of the known quantities a_{ij}, using relations (1.3) and (1.6)

$$\Lambda_{11} = k_1 a_{11}, \quad \Lambda_{12} = k_1 a_{12}, \quad \Lambda_{21} = k_2 a_{21}, \quad \Lambda_{22} = k_2 a_{22} \tag{1.12}$$

Since $k_1 > 0$, $k_2 > 0$, inequalities (1.7) yield

$$\det [\Lambda_{ij}] = k_1 k_2 \det [a_{ij}] > 0 \tag{1.13}$$

Using the properties of Cauchy-type integrals near the ends of the line of integration /7/ and applying them to Eq.(1.11) with boundary conditions (1.8), we obtain the asymptotic form of the vector function $\mathbf{f}(x)$, $\mathbf{f}'(x)$ near the zero

$$\mathbf{f}(x) \to 2\mathbf{N}\sqrt{x}; \quad \mathbf{f}'(x) \to \frac{\mathbf{N}}{\sqrt{x}}, \quad x \to 0 \tag{1.14}$$

where the vector $\mathbf{N}$ is not known, has to be determined, and is connected with the vector $\mathbf{K}$ of the stress intensity factors by the formula

$$\mathbf{K} = \begin{Bmatrix} K_{II} \\ K_I \end{Bmatrix} = \sqrt{2\pi}\, \mathbf{N} \tag{1.15}$$

The vector function $\mathbf{f}(x)$ is constant at infinity and is expressed in terms of the known vector of external stresses and a matrix inverse to Λ:

$$\mathbf{f}(\infty) = \Lambda^{-1} \mathbf{P}_\infty \tag{1.16}$$

2. ***Solution of the integrodifferential equation.*** The singular Eq.(1.11) belongs to the class of integrodifferential Prandtl equations and arises in many problems of mechanics and mathematical physics. The equation has been studied in sufficient detail for an unknown scalar function (e.g. /8/) and its simple solution was obtained in /3/. Below we construct the closed solution of the Prandtl vector Eq.(1.11).

We introduce the vector function $\mathbf{f}_0(\xi)$ according to the rule

$$\mathbf{f}_0(\xi) = \begin{cases} \mathbf{f}(\xi) - \mathbf{f}(\infty), & \xi > 0 \\ 0, & \xi < 0 \end{cases} \tag{2.1}$$

and rewrite Eq.(1.1) in the form

$$\Lambda \mathbf{f}_0(\xi) - \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathbf{f}_0'(t)}{t-\xi}\,dt = \begin{cases} 0, & \xi > 0 \\ \mathbf{b}(\xi), & \xi < 0 \end{cases} \tag{2.2}$$

where $\mathbf{b}(\xi)$ is an unknown vector function defined on the ray $\xi < 0$.

Let us find the complex vector functions

$$\mathbf{Q}_0^+(\omega) = \int_{-\infty}^{+\infty} \mathbf{f}_0(\xi)\, e^{i\omega\xi}\, d\xi, \quad \mathbf{Q}_1^+(\omega) = \int_{-\infty}^{+\infty} \mathbf{f}_0'(\xi)\, e^{i\omega\xi}\, d\xi \tag{2.3}$$

analytic in the upper half-plane $(\mathrm{Im}(\omega) > 0)$. Integrating the second equation of (2.3) by parts, we obtain the relation connecting $\mathbf{Q}_0^+$ with $\mathbf{Q}_1^+$:

$$\mathbf{Q}_1^+(\omega) = \Lambda^{-1}\mathbf{P}_\infty - i\omega \mathbf{Q}_0^+(\omega) \tag{2.4}$$

Using the Erdelyi lemma /9/, which is an analogue of the Watson lemma for the Fourier integrals, and taking into account the behaviour of the vector function $\mathbf{f}_0(x)$, $\mathbf{f}_0'(x)$ at the zero (see (1.14) and (2.1)), we can determine the asymptotic behaviour of the vector function $\mathbf{Q}_0^+$, $\mathbf{Q}_1^+$ at infinity:

$$\mathbf{Q}_1^+(\omega) \to \mathbf{N}\sqrt{\frac{\pi}{\omega}}\exp\left(i\frac{\pi}{4}\right), \tag{2.5}$$

$$\mathbf{Q}_0^+(\omega) \to -i\Lambda^{-1}\frac{\mathbf{P}_\infty}{\omega} - \frac{\mathbf{N}}{\omega}\sqrt{\frac{\pi}{\omega}}\exp\left(-i\frac{\pi}{4}\right), \quad \omega \to \infty$$

Applying to Eq.(2.2) a Fourier transformation in the coordinate ξ, and taking into account relations (2.3), we obtain

$$\Lambda \mathbf{Q}_0^+(\tau) + i\,\mathrm{sign}(\tau)\,\mathbf{Q}_1^+(\tau) = \mathbf{B}^-(\tau)$$

where τ is the affix of the point on the real axis of the complex ω plane and $\mathbf{B}^-(\tau)$ is the limiting value of the vector function $\mathbf{B}^-(\omega)$, which is a Fourier transform of the function $\mathbf{b}(\xi)$ analytic in the lower half-plane ω. Using (2.4), we can transform this equation into the problem of conjugation for a piecewise analytic vector function on the real axis τ (E is the unit matrix)

$$(\Lambda + \mathrm{E}\,|\tau|)\,\mathbf{Q}_1^+(\tau) = -i\tau\mathbf{B}^-(\tau) + \mathbf{P}_\infty \tag{2.6}$$

Let us consider the characteristic equation /10/ of the matrix Λ:

$$\det(\mathrm{E}\tau - \Lambda) = (\tau - \tau_1)(\tau - \tau_2) = 0 \tag{2.7}$$

The constants τ_1, τ_2 will be called its characteristic numbers. They are easily found and they satisfy the inequalities (which follow from (1.12), (1.13))

$$\tau_1\tau_2 = \det(\Lambda_{ij}) > 0, \quad \tau_1 + \tau_2 = (\Lambda_{11} + \Lambda_{22}) > 0$$

and ensure, as can be shown, a non-zero value of the determinant of the matrix accompanying the vector function $\mathbf{Q}_1^+$ in (2.6) for any value of τ.

Using this property, we shall write the matrix coefficient accompanying the vector function $\mathbf{Q}_1^+(\tau)$ in (2.6), in the form

$$\Lambda + \mathrm{E}\,|\tau| = [\mathrm{Z}^-(\tau)]^{-1}\,\mathrm{Z}^+(\tau) \tag{2.8}$$

where the square matrices $\mathrm{Z}^+(\omega), \mathrm{Z}^-(\omega)$ have, as their components, functions that are analytic, respectively, in the upper and lower half-plane of ω except, perhaps, at the point at infinity.

Below we shall assume that $\tau_1 \neq \tau_2$ (the non-equality of characteristic numbers). It can be shown that the matrices

$$\mathrm{Z}^+(\omega) = \mathrm{I}^+(\omega)\,(\Lambda + \omega\mathrm{E})^{1/2}, \quad [\mathrm{Z}^-(\omega)]^{-1} = (\Lambda + \omega\mathrm{E})^{1/2}\,\mathrm{I}^-(\omega) \tag{2.9}$$

$$\mathrm{I}^\pm(\omega) = \exp\left\{\frac{1}{2\pi i}\int_0^\omega (\mathrm{E}\ln\tau - \ln\Lambda)\left((\tau\mathrm{E} \mp \Lambda)^{-1} - (\tau\mathrm{E} \pm \Lambda)^{-1}\right)d\tau\right\}$$

defined in the whole complex plane ω with a cut along the negative part of the real axis possess the required properties.

In proving this fact we use, essentially, the properties of an analytic function of a matrix argument /10/. We know /10/ that for any analytic function $\vartheta(\zeta)$ and for any square matrix Λ of dimensions 2×2 with unequal characteristic numbers s_1, s_2, we can construct a function $\vartheta(\Lambda)$ of matrix argument according to the rule

$$\vartheta(\Lambda) = \frac{s_1\vartheta(s_2) - s_2\vartheta(s_1)}{s_1 - s_2} \mathrm{E} + \frac{\vartheta(s_1) - \vartheta(s_2)}{s_1 - s_2} \Lambda \tag{2.10}$$

In particular, the following relations hold:

$$\ln \Lambda = \frac{\tau_1 \ln(\tau_2) - \tau_2 \ln(\tau_1)}{\tau_1 - \tau_2} \mathrm{E} + \frac{\ln(\tau_1) - \ln(\tau_2)}{\tau_1 - \tau_2} \Lambda \tag{2.11}$$

$$(\tau\mathrm{E} \pm \Lambda)^{-1} = \pm \frac{(\tau_1 + \tau_2 \pm \tau)\mathrm{E} - \Lambda}{(\tau_1 \pm \tau)(\tau_2 \pm \tau)}$$

The matrix functions (2.9) which factorize the matrix coefficient in Eq.(2.6), represent a generalization of factorizing functions of the corresponding one-dimensional problem discussed in /3/.

Having used the properties of the matrix functions, we can obtain the following matrix inversion formulas for (2.9):

$$[\mathrm{Z}^+(\omega)]^{-1} = (\Lambda + \omega\mathrm{E})^{-1/2}\, \mathrm{I}^-(\omega), \quad \mathrm{Z}^-(\omega) = \mathrm{I}^+(\omega)\,(\Lambda + \omega\mathrm{E})^{-1/2} \tag{2.12}$$

Let us rewrite relation (2.6) using the matrices Z^+, Z^- satisfying the matrix Eq.(2.8), in the form

$$\mathrm{Z}^+(\tau)\, \mathbf{Q}_1^+(\tau) = \mathrm{Z}^-(\tau)\,(\mathbf{P}_\infty - i\tau\mathbf{B}^-(\tau))$$

The left-hand side of this equation represents the limit value of the vector function analytic in the upper half-plane of ω (except, perhaps, at the point at infinity), and the right-hand side is the limit value of the vector function analytic in the lower half-plane of ω. Therefore, according to the generalized Liouville theorem, the left and right sides of this equation represent a single vector function analytic in the whole complex plane ω except at the point at infinity. The vector function is a polynomial whose degree is determined by the behaviour of the left (or right)-hand side at infinity. Taking into account the asymptotic properties of the matrix-function $\mathrm{Z}(\omega)$:

$$\mathrm{Z}^+(\omega) \to \mathrm{E}\sqrt{\omega}\exp(-i\pi/4), \quad \omega \to \infty \tag{2.13}$$

and the asymptotic form (2.5), we obtain

$$\mathbf{Q}_1^+(\omega) = [\mathrm{Z}^+(\omega)]^{-1}\,\mathbf{C}_0 \tag{2.14}$$

The constant vector $\mathbf{C}_0$ can be expressed in terms of the unknown vector $\mathbf{N}$, from the condition that the asymptotic expressions (2.5) and (2.14) are identical:

$$\mathbf{C}_0 = \mathbf{N}\sqrt{\pi} = \mathbf{K}/\sqrt{2}$$

Thus we obtain, apart from an unknown vector $\mathbf{K}$, an explicit expression for the vector function $\mathbf{Q}_1^+(\omega)$ and hence, in accordance with (2.4), the vector function $\mathbf{Q}_0^+(\omega)$:

$$\mathbf{Q}_1^+(\omega) = [\mathrm{Z}^+(\omega)]^{-1}\,\mathbf{K}/\sqrt{2}, \quad \mathbf{Q}_0^+(\omega) = i\omega^{-1}(\mathbf{Q}_1(\omega) - \Lambda^{-1}\mathbf{P}_\infty) \tag{2.15}$$

The vector functions $\mathbf{f}_0(\xi)$, $\mathbf{f}_0'(\xi)$ are recovered from relations (2.15) using inverse Fourier transformations of (2.3):

$$\mathbf{f}_0(\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathbf{Q}_0^+(\omega)\, e^{-i\omega\xi}\, d\omega$$

The relation connecting the force vector $\mathbf{P}_\infty$ (1.11) with the stress intensity factors vector $\mathbf{K}$ (1.15) follows from the condition that the vector function $Q_0^+(\omega)$ (2.15) has no pole at zero. Since it follows from (2.12) and (2.15) that

$$[\mathrm{Z}^+(0)]^{-1} = \Lambda^{-1/2}, \quad \mathbf{Q}_1(0) = \Lambda^{-1/2}\mathrm{K}/\sqrt{2}$$

we obtain the required formula

$$\mathbf{K} = \sqrt{2}\Lambda^{-1/2}\mathbf{P}_\infty \tag{2.16}$$

and the matrix $\Lambda^{-1/2}$ is expressed in terms of E, Λ and the characteristic numbers τ_k in accordance with (2.10):

$$\Lambda^{-1/2} = \frac{\tau_1/\tau_1^{1/2} - \tau_2/\tau_1^{1/2}}{\tau_1 - \tau_2}\mathrm{E} - \frac{1}{\sqrt{\tau_1} + \sqrt{\tau_2}}\frac{\Lambda}{\sqrt{\tau_1\tau_2}} \tag{2.17}$$

It is clear that expression (2.16) holds for the values of p_∞, q_∞, which yield a non-negative value of the stress intensity factor K_I.

We have two asymptotic forms of the stress near the tip of the semi-infinite pseudomacrocrack, namely the near one (at the distances $r \ll \lambda^{-1}$), and the far one (at $r \gg \lambda^{-1}$). The near asymptotic form of the stress-strain state is identical with the usual asymptotic form typical for a crack in the anisotropic body discussed here, with the stress intensity factors K_{I} and K_{II}. The far asymptotic form is identical with the dislocation field possessing the components of the Burgers vector $\{a_{11}g(\infty)+a_{12}f(\infty),\ a_{21}g(\infty)+a_{22}f(\infty)\}$ (the values $f(\infty)$ and $g(\infty)$ are determined by relations (1.16)) and placed in an anisotropic medium. The stresses in this case decay as $\sim r^{-1}$.

In the case when elastic symmetry planes normal to coordinate axes exist, the non-diagonal terms of the matrices a_{ij} and Λ_{ij} vanish, and the problem splits into two independent problems, of normal separation and shear displacement respectively. In this case we obtain the following expressions for the stress intensity factors obtained from (2.16) and (2.17):

$$K_{\mathrm{I}}=p_\infty\sqrt{\frac{2}{\lambda_2}},\quad K_{\mathrm{II}}=q_\infty\sqrt{\frac{2}{\lambda_1}};\quad \lambda_2=2k_2\,\mathrm{Re}\left(i\,\frac{q_1\mu_2-q_2\mu_1}{\mu_2-\mu_1}\right),$$
$$\lambda_1=2k_1\,\mathrm{Re}\left(i\,\frac{p_1-p_2}{\mu_2-\mu_1}\right)$$

which is identical with the result obtained earlier in /3/.

REFERENCES

1. MARSHALL D.B., COX B.N. and EVANS A.G., The mechanism of matrix cracking in brittle-matrix fibre composites. Acta Met. 33, 11, 1985.
2. KAZ'MIN V.I., MILEIKO S.T. and TVARDOVSKII V.V., Fracture of a model composite with a ceramic matrix. Mekhanika Kompositnykh Materialov., 2, 1988.
3. TVARDOVSKII V.V., On the theory of pseudomacrocracks in an anisotropic body. Pt.1. A single pseudomacrocrack. Izv. Akad. Nauk SSSR, MTT, 2, 1991.
4. LEKHNITSKII S.G., Theory of Elasticity of an Anisotropic Body. Nauka, Moscow, 1977.
5. SHIFRIN E.I., A plane crack of normal separation whose edges interact linearly. Izv. Akad. Nauk SSSR, MTT, 5, 1988.
6. TVARDOVSKII V.V., Further results on rectilinear line cracks and inclusions in anisotropic medium. Teor. Appl. Fract. Mech. 13, 1990.
7. GAKHOV F.D., Boundary Value Problems. Nauka, Moscow, 1977.
8. KALANDIYA A.I., Mathematical Methods of Two-dimensional Elasticity. Nauka, Moscow, 1973.
9. FEDORYUK M.V., Asymptotics: Integrals and Series. Nauka, Moscow, 1987.
10. GANTMAKHER F.R., Matrix Theory. Nauka, Moscow, 1988.

Translated by L.K.